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Inelastic Electron Scattering from Nuclei and Single-Particle Excitations*

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Arguments are presented that the single-particle excitations in nuclei induced by inelastically scattered electrons dominate the inelastic cross section in large domains of the momentum transfer (q) and the energy loss (ω). The sum rules for fixed q and ω are derived which include the transverse electron-nucleus interactions to order q^2/M^2 (M being the nucleon mass). The results of the calculations of the inelastic cross section for C^{12} at $\theta=135°$ are discussed and compared with experimental data.

I. INTRODUCTION

THE inelastic electron scattering from nuclei has long been recognized as one of the most direct means to investigate the correlations in nuclear matter. Although the experimental data are still very scarce, there are quite a few theoretical papers on the subject.[1,2] The present paper deals with the region of the inelastic cross sections where the scattering from quasifree nucleons presumably dominates as has been suggested by several authors.[1,3–5] This part of the cross section (which looks like a big bump on the experimental curves) almost exhausts the sum rule for $\int_0^\infty d\omega\, \sigma(q,\theta,\omega)$, q, θ, ω being the momentum transfer, the scattering angle, and the energy loss, respectively. For that reason it should be understood first, since any other effects (e.g., the high-energy-loss tail due to the short-range dynamical correlations[2]) are small additions to this dominant effect.

There is also another aspect of this problem. If one investigates any production process in the field of a heavy nucleus [see Fig. 1(a)], for instance, production of μ pairs,[6] the Z vertex is the same as in the inelastic electron scattering process [Fig. 1(b)]. As long as one does not measure all the angles and momenta of the particles produced at the left-hand vertex, one can use the energy sum rule for the Z vertex (compare Refs. 6, 1). In any complete measurement, however, where all momenta and angles of produced particles are measured, one has to use a sum rule for Z vertex for fixed q and ω. In principle, one could replace the Z vertex in Fig. 1(a) by the experimentally measured vertex in Fig. 1(b), but this is not an easy task in practice and even an approximate formula which takes into account the dominant process at Z vertex may prove to be useful in analyzing processes of the 1(a) type.

The formulation of the problem given in Sec. II is quite general and, in principle, can be applied both to finite and infinite systems provided our complete set of states—used to define the single-particle creation and annihilation operators and then the charge and the current operators [compare Eqs. (10) and (14)]—satisfies proper boundary conditions. In the region of small energy losses where one detects well-defined nuclear levels, the shell-model wave functions seem to be the obvious choice for the complete set of states. On the other hand, the convenient set of states for calculating the high-energy-loss tail for large nuclei are plane waves (compare Ref. 2). Any other system would seem to be much more difficult to handle in this region. As we intend to reproduce the experimental curves above the region of the excitations of the well-defined nuclear levels and in order to have a consistent theoretical description of the bump region (considered here) and the high-energy-loss tail (considered in Ref. 2), we choose the plane waves as our complete set of states in the calculations presented in Sec. III.

II. THE SUM RULES

Recently, a sum rule has been investigated which gives the inelastic electron-nucleus cross section $\sigma(q,\omega,\theta)$ as a product of the Mott cross section and the ground-state expectation value of the density-density correlation function for the nucleus, provided one includes only the longitudinal electron-nucleon interaction. (See Ref. 2, henceforth called A.)

First, we want to generalize the above sum rule. In order to simplify the arguments, let us consider the electron-nucleus interaction in the first Born approximation[7] correct through order q^2/M^2. As is shown in

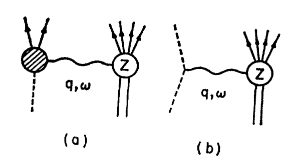

FIG. 1. (a) Production process on Coulomb field of a nucleus with Z protons. One sums over all nuclear excitations compatible with the momentum transfer q and energy loss ω. (b) The corresponding inelastic electron scattering.

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[1] K. W. McVoy and L. Van Hove, Phys. Rev. **125**, 1034 (1962).

[2] W. Czyż and K. Gottfried, Ann. Phys. (N. Y.) **21**, 47 (1963).

[3] J. E. Leiss and R. E. Taylor, Karlsruhe Photonuclear Conference, 1960, $F5$ (unpublished), and private communications. Experimental points shown in Figs. 5, 6, and 7 come from data privately communicated to the author by Dr. J. E. Leiss and Dr. R. E. Taylor.

[4] W. C. Barber, Ann. Rev. Nucl. Sci. **12**, 1 (1962).

[5] P. Bounin and G. R. Bishop, J. Phys. Radium **22**, 555 (1961).

[6] G. E. Masek, A. J. Lazarus, and W. K. H. Panofsky, Phys. Rev. **103**, 374 (1956).

[7] We accept here the point of view expressed in A that the breakdown of the Born approximation in scattering from large Z nuclei is not a serious obstacle which can be dealt with by the methods developed by Schiff (see A for detailed references).

Ref. 1, we have (henceforth we follow the notation of Ref. 1)

$$\begin{aligned}H'=\frac{4\pi e^2}{q_\mu^2}\langle u_f|F_1e^{iq_\mu x_\mu}&-\frac{F_1}{2M}[(\mathbf{p}\cdot\boldsymbol{\alpha})e^{iq_\mu x_\mu}+e^{iq_\mu x_\mu}(\mathbf{p}\cdot\boldsymbol{\alpha})]\\&-[(F_1+KF_2)/2M]i\boldsymbol{\sigma}\cdot(\mathbf{q}\times\boldsymbol{\alpha})e^{iq_\mu x_\mu}-(q_\mu^2/8M^2)\\&\times(F_1+2KF_2)e^{iq_\mu x_\mu}+[(F_1+2KF_2)/8M^2]i\boldsymbol{\sigma}\\&\cdot\{\mathbf{p}\times(\omega\boldsymbol{\alpha}-\mathbf{q})e^{iq_\mu x_\mu}-e^{iq_\mu x_\mu}(\omega\boldsymbol{\alpha}-\mathbf{q})\times\mathbf{p}\}|u_i\rangle,\quad(1)\end{aligned}$$

where $q_\mu^2=q^2-\omega^2$, q is the three momentum transfer, ω the energy loss, $\boldsymbol{\alpha}$ is the electron Dirac operator, $|u_i\rangle$ and $|u_f\rangle$ are the free-electron spinors, $\mathbf{p}$ and $\boldsymbol{\sigma}$ are the momentum and Pauli spin operators for the nonrelativistic nucleons, $F_1(q_\mu^2)$ and $F_2(q_\mu^2)$ are the standard nucleon form factors, and K is the static anomalous magnetic moment (in nuclear magnetons).

We use the following approximation for the nucleon form factors: $F_{1p}(q_\mu^2)=F_{2p}(q_\mu^2)=f(q_\mu^2)$ for the proton, and $F_{1n}(q_\mu^2)=0$, $F_{2n}(q_\mu^2)=f(q_\mu^2)$ for the neutron.

From (1) we get, summing over all nucleons inside of the nucleus, the following expression for the transition matrix element from the ground (energy E_0) to the excited state (energy E_n) of the nucleus[1]:

$$\begin{aligned}|M_{n0}|^2=\delta(\omega-E_n+E_0)&(4\pi e^2/q_\mu^2)^2f^2(q_\mu^2)\\&\times|(\langle u_f|u_i\rangle Q_{n0}-\langle u_f|\boldsymbol{\alpha}|u_i\rangle\cdot\mathbf{J}_{n0})|^2,\quad(2)\end{aligned}$$

where

$$Q_{n0}=\langle n|Q|0\rangle=\langle n|\sum_{j=1}^{A}[e_j+(q^2/8M^2)(e_j-2\mu_j)]e^{i\mathbf{q}\cdot\mathbf{r}_j}|0\rangle,$$

$$\begin{aligned}\mathbf{J}_{n0}=\langle n|\mathbf{J}|0\rangle=\langle n|\sum_{j=1}^{A}[(e_j/2M)&(\mathbf{p}_je^{i\mathbf{q}\cdot\mathbf{r}_j}+e^{i\mathbf{q}\cdot\mathbf{r}_j}\mathbf{p}_j)\quad(3)\\&+(\mu_j/2M)i\boldsymbol{\sigma}_j\times\mathbf{q}e^{i\mathbf{q}\cdot\mathbf{r}_j}]|0\rangle,\end{aligned}$$

where $|0\rangle$ and $|n\rangle$ are the nuclear ground and excited states, respectively, $\mathbf{r}_j$, $\mathbf{p}_j$, and $\boldsymbol{\sigma}_j$ are the position, momentum, and spin operators for the jth nucleon. Besides,

$$e_j=\tfrac{1}{2}(1+\tau_{zj}),$$

$$\mu_j=\tfrac{1}{2}(1+\tau_{zj})(1+K)-\tfrac{1}{2}(1-\tau_{zj})K=\tfrac{1}{2}(1+\tau_{zj})+\tau_{zj}K,$$

with $K\approx1.85$, so μ_j reproduces the neutron and proton magnetic moments.

If we square M_{n0} and sum and average over final and initial electron spin states, we get

$$\tfrac{1}{2}\sum_{\text{spins}}|M_{n0}|^2=\delta(\omega-E_n+E_0)(4\pi e^2/q_\mu^2)^2f^2(q_\mu^2)W_n,\quad(4)$$

where

$$\begin{aligned}(1+\cos\theta)^{-1}W_n&\\=Q_{n0}^*Q_{n0}&-(\omega/q)[(\hat{q}\cdot\mathbf{J}_{n0}^*)Q_{n0}+Q_{n0}^*(\hat{q}\cdot\mathbf{J}_{n0})]\\&+\tfrac{1}{3}(2\tan^2\tfrac{1}{2}\theta+1)\mathbf{J}_{n0}^*\cdot\mathbf{J}_{n0}-\tfrac{1}{2}(2\tan^2\tfrac{1}{2}\theta+1-3\omega^2/q^2)\\&\times[(\hat{q}\cdot\mathbf{J}_{n0}^*)(\hat{q}\cdot\mathbf{J}_{n0})-\tfrac{1}{3}(\mathbf{J}_{n0}^*\cdot\mathbf{J}_{n0})],\quad(5)\end{aligned}$$

$\hat{q}=\mathbf{q}/q$ and θ is the scattering angle as shown in Fig. 2.

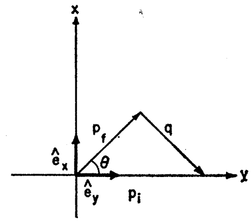

FIG. 2. Geometry of the electron scattering. p_i, p_f are the initial and final electron momenta, respectively. $\hat{e}_x$ and $\hat{e}_y$ are unit vectors.

The formula (5) is a starting point for constructing the sum rules for the inelastic cross section integrated over all energy losses. In fact, Eq. (5) can provide us with a formula (see, e.g., Ref. 1 for details and references to earlier papers) which gives

$$\sigma(q,\theta)=\int_0^\infty d\omega\,\sigma(\omega,q,\theta)\quad(6)$$

in terms of the ground-state expectation value of certain nuclear operators. We want, however, to follow the approach developed in A and construct the sum rule not for $\sigma(q,\theta)$, but rather for $\sigma(\omega,q,\theta)$. This can be accomplished with the help of the following formula. Let $A(0)$, $B(0)$ be two arbitrary operators at $t=0$ of the system with a Hamiltonian H such that $A_{n0}^*(0)\times B_{n0}(0)$ is real ($A_{n0}=\langle n|A|0\rangle$). Then the following identity holds:

$$\begin{aligned}\sum_n\delta(\omega-E_n+E_0)&A_{n0}^*(0)B_{n0}(0)\\&=-\frac{1}{\pi}\,\mathrm{Im}\int_{-\infty}^{+\infty}dt\,\frac{1}{i}e^{i\omega t}\langle0|T\{A^\dagger(t)B(0)\}|0\rangle,\quad(7)\end{aligned}$$

where $A^\dagger(t)=e^{iHt}A^\dagger(0)e^{-iHt}$ and $T\{\cdots\}$ is the time-ordered product. If $A_{n0}^*(0)B_{n0}(0)$ is not real, we can see from Eq. (5) that a term $A_{n0}(0)B_{n0}^*(0)$ is always associated with it, so one can work with the real sum $A_{n0}^*(0)B_{n0}(0)+A_{n0}(0)B_{n0}^*(0)$. Equation (7) is a straightforward generalization of the formula (2.18) of A (see also Ref. 8). From Eq. (5) we see that W can be expressed in terms of the imaginary parts of the Fourier transforms of

(a) density-density correlation function (scalar):

$$\begin{aligned}R(QQ;q\omega)=-\frac{1}{\pi}\,\mathrm{Im}\int_{-\infty}^{+\infty}&dt\,\frac{1}{i}e^{i\omega t}\\&\times\langle0|T\{Q^\dagger(t)Q(0)\}|0\rangle;\quad(8a)\end{aligned}$$

(b) density-current correlation function (vector):

$$\begin{aligned}R_l(QJ;q\omega)=-\frac{1}{\pi}\,\mathrm{Im}\int_{-\infty}^{+\infty}&dt\,\frac{1}{i}e^{i\omega t}\\&\times\langle0|T\{J_l^\dagger(t)Q(0)+Q^\dagger(t)J_l(0)\}|0\rangle;\quad(8b)\end{aligned}$$

[8] D. Pines, *The Many Body Problem* (W. A. Benjamin, New York, 1961).

(c) current-current correlation function (tensor):

$$R_{ll'}(JJ;q\omega)=-\frac{1}{\pi}\,\mathrm{Im}\int_{-\infty}^{+\infty}dt\,\frac{1}{i}e^{i\omega t}\times\langle 0|T\{J_l{}^\dagger(t)J_{l'}(0)+J_{l'}{}^\dagger(t)J_l(0)\}|0\rangle. \quad (8c)$$

Now we can write Eq. (5) as the ground-state expectation value and have thus a sum rule for $\sigma(q,\omega,\theta)$.

From Eq. (7) and Eqs. (8a,b,c), we get

$$(1+\cos\theta)^{-1}\sum_n\delta(\omega-E_n+E_0)W_n$$

$$=R(QQ;q\omega)-\frac{\omega}{q}\sum_l\hat{q}_lR_l(QJ;q\omega)$$

$$+\tfrac{1}{2}\sum_{ll'}\left[\left(\tan^2\tfrac{1}{2}\theta+\tfrac{1}{2}-\frac{\omega^2}{2q^2}\right)\delta_{ll'}+\left((3\omega^2/2q^2)-\tan^2\tfrac{1}{2}\theta-\tfrac{1}{2}\right)\hat{q}_l\hat{q}_{l'}\right]R_{ll'}(JJ;q\omega). \quad (9)$$

We call R, R_l, $R_{ll'}$ the response functions.

Obviously, if one is interested in the longitudinal interaction only (as it is the case in A), one is left with the $R(QQ;q\omega)$ function only. We may emphasize here that the sum rule (9) is exact in the sense that no approximations are made after the interaction Hamiltonian is assumed in the form shown by (1). This is not true in the case of the sum rule which gives the cross section integrated over energy losses. In order to prove this sum rule, one uses the closure approximation, and the ω and ω^2 in (5) have to be approximated by some $\langle\omega\rangle_{\mathrm{av}}$ and $\langle\omega^2\rangle_{\mathrm{av}}$ estimated independently (compare Ref. 1). Equation (9) is the starting point of our discussion.

We want to use the second quantization formulation in discussing R functions. We choose certain complete system of states $|\alpha\rangle$, where α stands for a complete set of quantum numbers for a single nucleon (they may be, for example, spin, isospin, angular momentum, and energy). For Q and $\mathbf{J}$ operators we then get

$$Q=\sum_{\alpha\beta}Q(\alpha\beta)a_\alpha{}^\dagger a_\beta,\quad \mathbf{J}=\sum_{\alpha\beta}\mathbf{J}(\alpha\beta)a_\alpha{}^\dagger a_\beta, \quad (10)$$

where $a_\alpha{}^\dagger a_\beta$ are the Fermi particle creation and annihilation operators and $Q(\alpha\beta)$ and $\mathbf{J}(\alpha\beta)$ are the matrix elements of the single-particle operators which appear in Eq. (3). If we choose for α spin, isospin, and momentum, which is the most convenient choice for an infinite system, we have the following expressions for Q and $\mathbf{J}$ operators:

$$Q=\sum_{\tau_1\tau_2}\sum_{s_1s_2}\sum_k\delta_{s_1s_2}\delta_{\tau_1\tau_2}\left\{\tfrac{1}{2}(1+\tau_z)-\frac{q^2}{16M^2}[1+\tau_z(1+4K)]\right\}a_{s_1\tau_1}{}^\dagger(k+q)a_{s_2\tau_2}(k),$$

$$\mathbf{J}=\sum_\tau\sum_{s_1s_2}\sum_k\left\{\delta_{\tau_1}\left[\frac{\delta_{s_1s_2}}{2M}(2\mathbf{k}+\mathbf{q})+\frac{1+K}{2M}i\boldsymbol{\varrho}_{s_1s_2}\right]-\delta_{\tau-1}\frac{K}{2M}i\boldsymbol{\varrho}_{s_1s_2}\right\}a_{s_1\tau}{}^\dagger(k+q)a_{s_2\tau}(k), \quad (11)$$

where $\boldsymbol{\varrho}=(\boldsymbol{\sigma}\times\mathbf{q})_{s_1s_2}$, s_1, s_2 being the z-component spin quantum numbers. If we use the reference system shown in Fig. 2 with xy plane chosen to be the electron scattering plane and the spins of the nucleons are quantized along the z axis, we get

$$\boldsymbol{\varrho}=\begin{pmatrix}-q_y\hat{e}_x+q_x\hat{e}_y, & (q_y+iq_x)\hat{e}_z\\ (q_y-iq_x)\hat{e}_z, & q_y\hat{e}_x-q_x\hat{e}_y\end{pmatrix}. \quad (12)$$

Now we express all R's in terms of *only one* two-particle Feynman propagator defined as

$$K(\alpha\beta\gamma\delta;t)=\langle 0|T\{a_\beta{}^\dagger(t)a_\alpha(t)a_\gamma{}^\dagger(0)a_\delta(0)\}|0\rangle,$$

$$iK(\alpha\beta\gamma\delta;\omega)=\int_{-\infty}^{+\infty}dt\,e^{i\omega t}K(\alpha\beta\gamma\delta;t), \quad (13)$$

namely,

$$R(QQ;q\omega)=-\frac{1}{\pi}\,\mathrm{Im}\sum_{\alpha\beta\gamma\delta}Q(\alpha\beta)Q^*(\gamma\delta)\times K(\alpha\beta\gamma\delta;\omega), \quad (14a)$$

$$R_l(QJ;q\omega)=-\frac{1}{\pi}\,\mathrm{Im}\sum_{\alpha\beta\gamma\delta}[J_l{}^*(\alpha\beta)Q(\gamma\delta)+Q^*(\alpha\beta)J_l(\gamma\delta)]K(\alpha\beta\gamma\delta;\omega), \quad (14b)$$

$$R_{lm}(JJ;q\omega)=-\frac{1}{\pi}\,\mathrm{Im}\sum_{\alpha\beta\gamma\delta}[J_l{}^*(\alpha\beta)J_m(\gamma\delta)+J_m{}^*(\alpha\beta)J_l(\gamma\delta)]K(\alpha\beta\gamma\delta;\omega). \quad (14c)$$

Formulas (9) and (14) reduce, therefore, the problem of calculating the inelastic cross section to evaluation of one Feynman propagator $K(\alpha\beta\gamma\delta;\omega)$ and can, in principle, be applied to small-energy-loss cross sections, where the single-particle or collective levels are excited, as well as to large energy losses, where no well-defined nuclear states seem to be excited. In fact, a completely analogous formulation was used in Ref. 9 to study the giant dipole resonance excitation by electromagnetic radiation. Its properties were described in terms of properties of $K(\alpha\beta\gamma\delta;\omega)$ propagator.

[9] W. Czyż, Acta Phys. Polon. **20**, 737 (1961).

From Eqs. (9)–(14) one sees that the introduction of the transverse interactions does not complicate the calculations of the response function in any essential way. In the computations of the response function for the hard-core gas model (see A) we already had to use an electronic computer, so the introduction of the known $\mathbf{J}(\alpha\beta)$ functions given by Eq. (11) and Eqs. (14) is not going to cause any serious troubles.[10]

III. QUASIELASTIC ELECTRON SCATTERING FROM NUCLEI

In a typical experimentally measured inelastic cross section (see, e.g., Fig. 7) we would like to distinguish the following three parts of the cross-section curve which we shall henceforth call (a), (b), and (c):

(a) corresponds to the region of ω where well-defined nuclear levels are excited (small ω's);
(b) corresponds to the region of ω where one sees the characteristic broad bump (large ω's);
(c) corresponds to the tail of the bump (very large ω's).

In A the (c) part was investigated and the conclusion reached that it represents the inelastic electron scattering from fluctuations in the nuclear density distribution. The calculations of the cross section for the hard-core Fermi gas model was also given there. In order to define precisely the region (c), however, one has to understand much better the bump region (b) and the purpose of the considerations presented here is just to describe that part of the cross section. Even from very simple considerations which assume that the nucleus is the free degenerate Fermi gas, one finds the existence of the broad bump roughly of the right shape (see Fig. 8 of A). Starting with this observation, we are going to calculate that part of the cross section which comes from the sum of all the single-particle excitations compatible with the Pauli principle. We believe that this is the dominant process in the (b) region.

One may add here that the calculations of the sum rule $\int_0^\infty d\omega\, \sigma(q,\omega,\theta)$ seem to support this point of view also. They consistently indicate (see Refs. 1, 11, and 12) that $\int_0^\infty d\omega\, \sigma(q,\omega,\theta)$ depends very insignificantly on the dynamical nucleon-nucleon correlations, i.e., it is given by the properly antisymmetrized shell-model ground-state wave function. As we expect the dynamical short-range correlations to contribute as much to the excited final states as to the ground-state fluctuations, we consider the small contribution of the ground-state fluctuations, established by the $\int_0^\infty d\omega\, \sigma(q,\omega,\theta)$ sum rule, as an argument in favor of the dominance of the single-particle excitations in the (b) region, which almost exhaust the sum $\int_0^\infty d\omega\, \sigma(q,\omega,\theta)$ after all.

If we accept this point of view, we can picture our

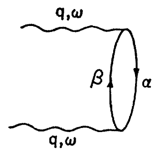

FIG. 3. The graph which dominates in the big-bump region of the inelastic cross section. It amounts to the quasifree electron-nucleon scattering.

process by a graph shown in Fig. 3. (For more details about these graphs see Refs. 2 and 9.)

The corresponding expression for ImK of Eq. (13) is

$$\mathrm{Im}K_0(\alpha\beta\gamma\delta;\,\omega) =\delta_{\alpha\gamma}\delta_{\beta\delta}n(\alpha)(1-n(\beta))\delta(\omega-E(\beta)+E(\alpha))\,, \quad (15)$$

where $n(\alpha)$ is the occupation number for the α state. If α represents a plane wave with momentum k, $n(\alpha)$ is the free-Fermi-gas momentum distribution.

If we allow for a momentum distribution $n(k)$ and the single-particle energies $E(k)$ differ from those of the free Fermi gas, we get the formulas which we shall call the impulse approximation. Although they do not form any consistent approximation to the problem, nevertheless, they prove to be useful in various physical problems, and we expect them to work pretty well in the (b) region of the inelastic cross section. In the impulse approximation, after performation of the summations and integrations indicated in (11) and (14), we get the following expression for the inelastic cross section:

$$\sigma(q,\omega,\theta)=f^2(q_\mu^2)\left(\frac{e^2}{2p_i}\right)^2\frac{1}{\sin^4\frac{1}{2}\theta}\sum_n\delta(\omega-E_n+E_0)W_n\,, \quad (16)$$

where p_i is the incident electron momentum and W evaluated from (9) gives

$$(1+\cos\theta)^{-1}\sum_n\delta(\omega-E_n+E_0)W_n =\frac{V}{(2\pi)^3}I\{A(q,\omega)+B(q,\omega)\tan^2\tfrac{1}{2}\theta\}\,, \quad (17)$$

where V is the volume of the nucleus,

$$I=\int d^3k\, n(k)[1-n(|k+q|)] \times\delta(\omega-E(k+q)+E(k))\,, \quad (18)$$

$$A(q,\omega)=2\left\{1-\frac{\omega}{M}\left(\frac{2}{q}\Omega_1+1\right) +\frac{1}{M^2}\left[\left(\frac{G}{2}-\Omega_2\right)\left(1-\frac{\omega^2}{q^2}\right)+\Omega_2\right] +\frac{\omega^2}{M^2}\left(\frac{\Omega_1}{q}-2.63\right)+1.71\frac{q^2}{M^2}\right\}, \quad (19)$$

$$B(q,\omega)=(2/M^2)\{G+5.77q^2-\Omega_2\}\,. \quad (20)$$

[10] This was pointed out to the author by Dr. A. Goldberg.
[11] S. D. Drell and C. L. Schwartz, Phys. Rev. 112, 568 (1958).
[12] W. E. Drummond, Phys. Rev. 116, 183 (1959).

In (19) and (20) the value $K=1.85$ was used.

$$\Omega_1=I^{-1}\int d^3k\, n(k)[1-n(|k+q|)] \times k\cos\phi\,\delta(\omega-E(k+q)+E(k)), \quad (21a)$$

$$\Omega_2=I^{-1}\int d^3k\, n(k)[1-n(|k+q|)] \times k^2\cos^2\phi\,\delta(\omega-E(k+q)+E(k)), \quad (21b)$$

$$G=I^{-1}\int d^3k\, n(k)[1-n(|k+q|)] \times k^2\delta(\omega-E(k+q)+E(k)), \quad (21c)$$

where $\phi=\sphericalangle(k,q)$. $E(k)$ is the energy of a single-particle excitation as a function of its momentum. The volume V in (17) we get from the normalization equation which for equal number of neutrons and protons is

$$4\int n(p)\frac{d^3p}{(2\pi)^3/V}=2Z. \quad (22)$$

In the case of the free Fermi gas, obviously $E(k)=k^2/2M$ and Ω_1 and Ω_2 are no longer integrals. In fact,

$$\Omega_1=\Omega=M\omega/q-\tfrac{1}{2}q,\quad \Omega_2=\Omega^2, \quad (23)$$

in this case, and $n(k)$ is the well-known Fermi step function. The integrals I and G are easy to evaluate:

$$I=\pi(M/q)[k_F^2-(M\omega/q-\tfrac{1}{2}q)^2] \quad \text{for } \omega>\omega'' \qquad (24a)$$
$$=2\pi(M^2/q)\omega \quad \text{for } \omega<\omega'',$$

$$G=\tfrac{1}{2}[k_F^2+(M\omega/q-\tfrac{1}{2}q)^2] \quad \text{for } \omega>\omega'' \qquad (24b)$$
$$=k_F^2-M\omega \quad \text{for } \omega<\omega'',$$

where $\omega''=qk_F/M-q^2/2M$. For V we get from (17)

$$V=3\pi^2Z/k_F^3. \quad (25)$$

As we believe that the (b) part of the inelastic electron cross section is dominated by the quasielastic electron-nucleon scattering, the above formulas can give us qualitatively the relative importance of the longitudinal and transverse interactions. If we normalize the Coulomb contribution to 1, we can write the transverse contribution in the form

$$\delta(q,\omega,\theta)=\alpha(q,\omega)+\beta(q,\omega)\tan^2\tfrac{1}{2}\theta, \quad (26)$$

where α and β are read off (19)

$$\alpha(q,\omega)=-\frac{\omega}{M}\left(\frac{2\Omega}{q}+1\right)+\frac{1}{M^2}\left[\left(\frac{G}{2}-\tfrac{3}{2}\Omega^2\right)\left(1-\frac{\omega^2}{q^2}\right)+\Omega^2\right]+\frac{\omega^2}{M^2}\left(\frac{\Omega}{q}-2.63\right)+1.71\frac{q^2}{M^2},$$

$$\beta(q,\omega)=(1/M^2)(G+5.77q^2-\Omega^2).$$

The α and β coefficients are shown in Fig. 4. This is, of course, a very crude estimation and gets worse with increasing q as our approximate interaction Hamiltonian (1) is supposed to work well if $q^2/M^2\ll 1$. Nevertheless, one can see that the transverse interactions are very important even for small θ, provided the momentum transfer is of the order of k_F. Besides, due to complicated interference effects, α becomes negative for large energy losses. As β is positive, it results in a destructive interference of the transverse interaction contributions and for certain θ's we may have only the Coulomb interaction contributing. The curves α and β in Fig. 4 stop at $\omega=\omega_c=qk_F/M+q^2/2M$, where I, thus, the cross section, becomes zero. One can accept, however, that even for ω slightly bigger than ω_c, the relative importance of the longitudinal and transversal contributions is roughly given by (26). In such a case the moral of the present estimation would be that one has to include the transverse interactions in any realistic calculations dealing with short-range correlations (which according to A dominate for $\omega>\omega_c$), even in the case of small-angle inelastic scattering. On the other hand, however, one may again expect for certain angles θ a destructive interference to occur which would make the transverse interaction contribution negligible.

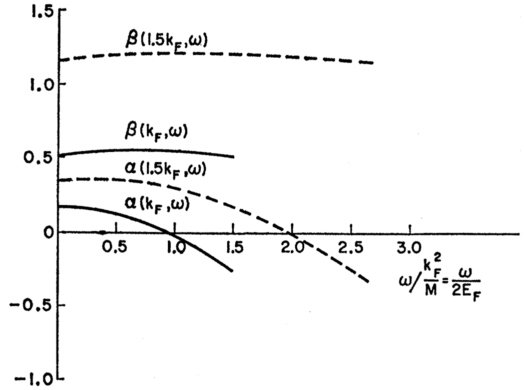

FIG. 4. α and β coefficients of (26) versus energy loss for two momentum transfers $q=k_F$, $q=1.5k_F$, where the Fermi momentum $k_F=280$ MeV. The Coulomb contribution is normalized to 1.

Unfortunately, there is no experimental data on inelastic electron scattering from large nuclei which would make it possible to confront our impulse approximation formulas (18)–(21) with experiment. In order to see, however, that the (b) region is indeed dominated by the single-particle excitations, we compare our impulse approximation cross section with experimental data on inelastic electron scattering from C^{12}. For that purpose we use the momentum distribution as given by the oscillator well model of C^{12} nucleus,

$$n(p)=[1+\tfrac{4}{3}(p/p_0)^2]e^{-(p/p_0)^2}. \quad (27)$$

In this case

$$V=\tfrac{4}{3}(\pi^{3/2}/p_0^3)Z. \quad (28)$$

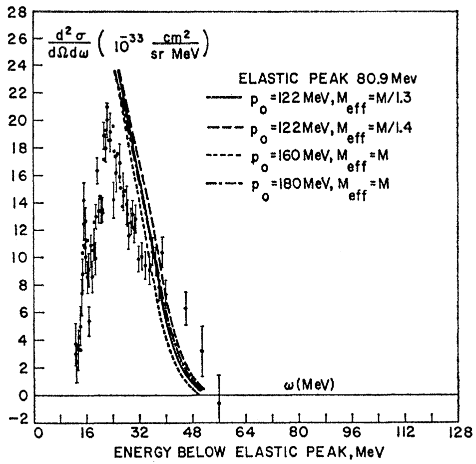

FIG. 5. Comparison of impulse approximation calculations for C^{12} with measurements of Leiss and Taylor (Ref. 3). The energy of incident electrons is 80.9 MeV. The scattering angle is 135°. The nucleon form factors are taken from Ref. 13.

The results for $E(k)=k^2/2M$ are shown in Figs. 5, 6, and 7. The curve begins at $\omega=25$ MeV as we want to be outside of the region where individual well-defined states are seen and our simple mechanism is certainly not applicable. Bearing in mind that we have chosen plane waves as our basic complete set of states which are suitable for nuclear matter calculations rather than for small nuclei like C^{12}, the agreement with experiment is rather good. We would consider it encouraging to try to fit any forthcoming data on the (b) region of the inelastic electron cross section from large nuclei in terms of single-particle excitations.

We have also done a bit more realistic calculations of the C^{12} cross sections (Figs. 5, 6, and 7) using the root-mean-square radius of C^{12} nucleus determined by the elastic electron scattering[13] which gives $p_0\cong 122$ MeV in (27). Then the introduction of some average effective mass of the order $0.7M$–$0.77M$ improves considerably the agreement with experiment. Introduction of an average effective mass is probably not a bad approximation, if ω is not too large. For large ω's the fact that $M_{\text{eff}}(k)$ increases with k turns out to be important, as indicated by the discussion below. Another point which one should keep in mind is that although the plane waves which we use as the basic complete set of states do not seem to be a good zeroth approximation for a light nucleus like C^{12}, the process of summation over all numerous possible single-particle excitations may turn out to be rather insensitive to the choice of the basic set of states.

We would like also to point out that the cross section at $\theta=135°$ and 148.5 MeV of incident electron energy (Fig. 7) is sensitive to the assumed magnetic moments of the nucleons inside of the nucleus. In fact, the dominant term in (17) is $B(q,\omega)\tan^2\frac{1}{2}\theta$, which depends on K roughly like $(0.5+K+K^2)$. The cross section is evaluated for $K=1.85$ but, e.g., 10% changes in K result approximately in 10% changes in the cross section.

Some general remarks are in order here. As was already said, the impulse approximation is not a consistent approximation in general. In the case of infinite nuclear matter, however, one can do a better job dressing both the particle and the hole lines (see Fig. 3) in some more or less consistent way. First, the integrals in (17)–(19) now have the form

$$I=-\frac{1}{\pi}\,\text{Im}\left\{\frac{1}{2\pi i}\int d^3k\int_{-\infty}^{+\infty}d\epsilon\,G_1(k,\epsilon)G_1(k+q,\,\epsilon+\omega)\right\},\tag{29a}$$

$$\Omega_1=-(\pi I)^{-1}\,\text{Im}\left\{\frac{1}{2\pi i}\int d^3k\,k\cos\phi\times\int_{-\infty}^{+\infty}d\epsilon\,G_1(k,\epsilon)G_1(k+q,\,\epsilon+\omega)\right\},\tag{29b}$$

$$\Omega_2=-(\pi I)^{-1}\,\text{Im}\left\{\frac{1}{2\pi i}\int d^3k\,k^2\cos^2\phi\times\int_{-\infty}^{+\infty}d\epsilon\,G_1(k,\epsilon)G_1(k+q,\,\epsilon+\omega)\right\},\tag{29c}$$

$$G=-(\pi I)^{-1}\,\text{Im}\left\{\frac{1}{2\pi i}\int d^3k\,k^2\times\int_{-\infty}^{+\infty}d\epsilon\,G_1(k,\epsilon)G_1(k+q,\,\epsilon+\omega)\right\},\tag{29d}$$

where $G_1(p,\epsilon)$ is the exact one-particle propagator. $G_1(p,\epsilon)$ has the well-known integral representation (see,

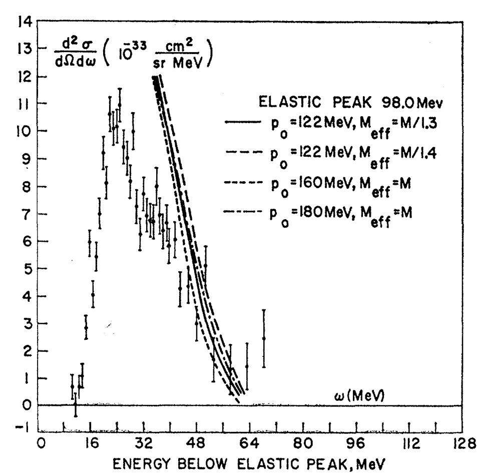

FIG. 6. The same as in Fig. 5, except the incident electron energy is 98.0 MeV.

[13] R. Herman and R. Hofstadter, *High-Energy Electron Scattering* (Stanford University Press, Stanford, California, 1960).

e.g., Ref. 14)

$$G_1(p,\epsilon)=\int_{-\infty}^{E_F} dE\frac{\sigma_-(p,E)}{\epsilon-E-i\eta}+\int_{E_F}^{\infty} dE\frac{\sigma_+(p,E)}{\epsilon-E+i\eta}, \quad (30)$$

where E_F is the Fermi energy. Thus,

$$-\frac{1}{\pi}\,\mathrm{Im}\left\{\frac{1}{2\pi i}\int_{-\infty}^{+\infty} d\epsilon\, G_1(k,\epsilon)G_1(k+q,\,\epsilon+\omega)\right\}$$
$$=\int_{E_F}^{\infty} dE\,\sigma_+(k+q,\,E)\sigma_-(k,\,E-\omega). \quad (31)$$

If the independent-particle description of nuclear matter is valid in the zeroth approximation (a fact which is commonly accepted), σ_+ and σ_- are strongly peaked (for $p>k_F$ and $p<k_F$, respectively) functions of the form

$$\sigma_+(p,E)=\frac{\Gamma_+}{[E-E(p)]^2+(\pi\Gamma_+)^2},$$
$$\sigma_-(p,E)=\frac{\Gamma_-}{[E-E(p)]^2+(\pi\Gamma_-)^2}, \quad (32)$$

where Γ's are small in comparison to $[E-E(p)]$ and depend, in general, on p and E (see, e.g., Ref. 14). $E(p)$ is here the energy of a dressed particle (hole) with momentum p and Γ^{-1} is its lifetime. Assuming Γ's negligible, we get

$$-\frac{1}{\pi}\,\mathrm{Im}\left\{\frac{1}{2\pi i}\int_{-\infty}^{+\infty} d\epsilon\, G_1(k,\epsilon)G_1(k+q,\,\epsilon+\omega)\right\}$$
$$\approx\delta[\omega-E(k+q)+E(k)],\quad |k+q|>k_F>k. \quad (33)$$

Equation (33) essentially reproduces our impulse approximation, except we cannot now change the momentum distribution at will, since the exact relation

$$n(p)=\int_{-\infty}^{E_F} dE\,\sigma_-(p,E)$$

tells us that assuming Γ_- very small we have to have $n(p)$ which is virtually the free Fermi gas step function. The energy-momentum relation $E=E(k)$, however, can be very different from the free-particle one $(E=k^2/2M)$ as the Brueckner type of calculations show.[14] The difference is usually lumped into the effective mass which comes out to be lighter than the free mass and, deep inside of the nuclear matter, is presumably $M_{\mathrm{eff}}\approx 0.5M-0.7M$.

The existence of the effective mass results in drastic changes of the q,ω region where the graph of Fig. 3

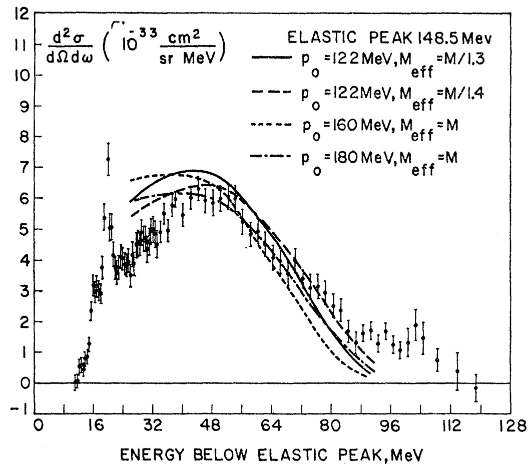

FIG. 7. The same as in Fig. 5, except the incident electron energy is 148.5 MeV.

dominates. For the free Fermi gas the Fig. 3 graph gives no contribution for $\omega>\omega_c=k_Fq/M+q^2/M$ and for such ω's we expect the cross section to be given by the scattering of the electron from the dynamical fluctuations of the nuclear matter (see A). We would expect, however, that in case of large ω one should use the full mass of the nucleon only in the particle state (labeled β in Fig. 3), since the particle is well above the Fermi sea. For the hole state (labeled α) which is inside of the Fermi sea, the effective mass should be used and the energy conservation reads

$$\omega=(\mathbf{k}+\mathbf{q})^2/2M-\mathbf{k}^2/2M_{\mathrm{eff}}. \quad (34)$$

A straightforward calculation shows us that, for $\omega>\tilde{\omega}$,

$$\tilde{\omega}=q^2/2\Delta M,\quad \Delta M=M-M_{\mathrm{eff}}. \quad (35)$$

We cannot satisfy the energy conservation any more because the particles "floating" inside of nuclear matter are lighter than the particles lifted far above k_F. In order to see how important this effect may be, let us assume $M_{\mathrm{eff}}\approx 0.5M$ and $q\approx k_F$. Then we get $\tilde{\omega}\approx k_F^2/M$, whereas $\omega_c\approx\frac{3}{2}k_F^2/M$. For $k_F\approx 280$ MeV, we get $\tilde{\omega}\approx 87$ MeV and $\omega_c\approx 131$ MeV; so the "effective mass effect" kills the dominance of the Fig. 3 diagram 44 MeV *below* ω_c, and anything seen above $\tilde{\omega}$ would already be the scattering from the dynamical fluctuation of the nuclear matter (see A). Of course, this is just an oversimplified example and in more realistic calculations one should introduce the Γ's and use the overlap integral (31) instead of the conserving energy δ function (33). Then the formulas (16)–(21) can again be used to compute the cross section provided we make the following replacement:

$$n(k)[1-n(k+q)]\delta(\omega-E(k+q)+E(k))\Rightarrow\int_{E_F}^{\infty} dE\frac{\Gamma_+\Gamma_-}{[(E-E(k+q))^2+(\pi\Gamma_+)^2][(E-\omega-E(k))^2+(\pi\Gamma_-)^2]}. \quad (36)$$

[14] J. Goldstone, in Proceedings of the International School of Physics "Enrico Fermi," 1960 (unpublished); in *Nuclear Spectroscopy* (Academic Press, Inc., New York, 1960), p. 182.

We see that the bigger the Γ's, the less pronounced is the effect mentioned above. From the known order of magnitude of the imaginary part of the optical potential, one would estimate that $\tilde{\omega}$ is smeared out over a 10–20-MeV energy interval. At any rate it seems that the optical parameters of the dispersive nuclear medium which "dress" the particle and the hole may be as important in defining the region of dominance of the "quasielastic" scattering as the Fermi momentum k_F which gives ω_c (compare A). An alternative way of analyzing the (b) region of the inelastic electron cross sections would then be in terms of the replacement (36), where the $E(k)$ function could be taken from a Brueckner type of calculation and Γ's left as free parameters. In any case, the above considerations indicate that the measurements of the (b) region give a direct access to the otherwise hard to measure functions $E(k)$.

CONCLUSIONS

(1) The paper presents arguments that in the region of a big bump in the inelastic electron-nucleus cross section the graph shown in Fig. 3 (i.e., quasielastic scattering) dominates.

(2) The sum rule including all the transverse interactions to order q^2/M^2 is given. The importance of the transverse interactions is estimated and found important for q and ω of order k_F and E_F, respectively.

(3) From the example of inelastic scattering from C^{12} nucleus at $\theta = 135°$, one sees that a very large percentage of the cross section is due to the magnetic moments of the nucleons inside of C^{12} nucleus. The cross section for large q's (Fig. 7) is very sensitive to the magnitude of the magnetic moments. Figure 7 shows the agreement with experiment for very reasonable values of M_{eff} and p_0 and for magnetic moments equal to those of the free nucleons. So, one may conclude that the magnetic moments of the nucleons inside of nuclei cannot be appreciably different from the free nucleon ones. Unfortunately, this argument is not model-independent.

(4) There exist some complicated interference effects between different parts of the transition amplitude (longitudinal-transverse, transverse-transverse). There are regions of the q, ω plane where they act constructively and regions where they act destructively (compare Fig. 4). Consequently, for certain angles θ the transverse effects cancel almost completely and only the Coulomb interaction contributes.

(5) In order to define precisely the region of the q, ω plane where the dynamical correlations dominate, one has to study extensively, both experimentally and theoretically, the bump region. The finite size effects are not the only ones which make the estimations based on the free Fermi gas model unreliable (compare A). In this paper, it is argued that the dispersive effects of the nuclear medium are also very important in defining that region. In particular, the shape of the bump (e.g., the position of its maximum) depends on the energy-momentum relation $E(k)$ of the single-particle excitations. Thus, the analysis of the forthcoming experimental data on the inelastic electron scattering will presumably give a direct access to the otherwise hard to measure function $E(k)$.

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